Math 656 • Midterm Examination • March 27, 2015 • Prof. Victor Matveev

No electronic devices allowed. Please show **all solution steps** to receive full credit

1) **(14pts)** Find **all values** of z in polar or Cartesian form, and plot them as points in the complex plane:

(a) $z = \dfrac{\left(1+\sqrt{3}i\right)^{1/2}}{i^{\,1/3}}$ **(b)** $z = \cosh^{-1}(i)$ (start from the definition of cosh *z*)

2) **(13pts)** Sketch the image of a square defined by vertices *z*=*i*, *z*=0, *z*=1 and z=1+*i* under the mapping $w = \dfrac{1+i}{\sqrt{2}}\left(\overline{z}\right)^2$. Hint: treat this mapping as a sequence of 3 simple transformations.

3) **(21pts) Use an appropriate method** to calculate each integral over the indicated contour

(a) $\oint\limits_{|z-1|=2} \dfrac{\cos z\ dz}{z^2\left(z^2-4\right)}$ Integral over a circle of radius 2 around point z=1

(b) $\oint\limits_{|z|=1} \dfrac{z\,dz}{\left(e^z-1\right)^2}$ Integral over a circle of radius 1 (hint: find a couple dominant terms in the Laurent series)

(c) $\int\limits_C \cosh\left(\log_\pi \overline{z}\right)dz$ C = semi-circle in the right half-plane of radius 1 centered at the origin and connecting point –*i* to point +*i* . Use the principal branch of the logarithm $\log_\pi z$ defined by $-\pi \le \arg_\pi z < \pi$.

4) **(13pts)** Find an upper bound for $\left|\int\limits_C \dfrac{e^{iz}\log_o z\ dz}{z^2+4}\right|$, where the integration contour *C* is a straight line connecting point *z*=*i* to point *z*=1 (assume $0 \le \arg_o z < 2\pi$). Hint: treat each of the three factors separately.

5) **(13pts)** Find the first three dominant terms in the Taylor series for function $f(z) = \dfrac{1}{1+\log_\pi z}$ near point *z*=1; indicate where the full series would converge. Use the branch of logarithm $-\pi \le \arg_\pi z < \pi$.

==================== **Pick any two problems between 6, 7, 8** ====================

6) **(13pts)** Suppose a given Laurent series $\sum\limits_{k=-\infty}^{+\infty} c_k\left(z-z_o\right)^k$ has a maximal domain of convergence described by $0 < r < |z-z_o| < R$. Does the principle part of this series converge anywhere outside this ring? What about the positive-power part, $\sum\limits_{k=\mathbf{0}}^{+\infty} c_k\left(z-z_o\right)^k$?

7) **(13pts)** Find ***all*** series representations centered at $z_o=i$ for function $f(z) = \dfrac{1}{(z-i)^2(z+1)}$, and indicate their respective domains of convergence. Note: partial fractions are not needed in this problem (this problem is somewhat easier than the similar homework problem that we had).

8) **(13pts)** Make a **rough sketch** of the domain of convergence of the series $\sum\limits_{k=0}^{+\infty} \dfrac{\left(\log_\pi z\right)^k}{k^2}$